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Fatigue crack initiation in glassy plastics in high strain fatigue tests

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High strain fatigue tests were carried out on polycarbonate and polyvinylchloride. The roles of crazes and shear bands in the initiation and growth of fatigue cracks were investigated. It was found that a minimum surface strain of 2.7% was required for crazes to form in polycarbonate within a couple of hours, and that crack propagation from these crazes was hindered by the shear bands that form at their ends, and the presence of neighbouring cracks. A stress analysis of the formation of shear bands at the end of crazes was carried out, to obtain values for the shear stress acting on the shear bands.

1. Introduction

There have been recent reviews of the mechanisms of fatigue in initially uncracked [1] and initially cracked [2] polymer specimens. Both emphasize the role of crazing in the fatigue of some glassy polymers, although it is stated [1] that at 298 K polycarbonate (PC) does not craze. There is, however, no attempt to link the fracture processes seen on the fatigue fracture surface with the pattern of crazes which may have appeared earlier on the free surface of the stressed polymer. We would like to know whether the largest craze on the surface initiates the fatigue crack irrespective of its position, or whether the mutual positioning of neighbouring crazes is important. In the course of fatigue tests on PC [3] and polyvinylchloride (PVC) [4] craze interactions were observed. The results with PC were inconsistent, so were repeated to be more certain of the stresses and strains involved. The fracture processes in these two polymers are described, then an attempt is made to analyse some of them using a recently developed stress analysis method [5].

2. Experimental details

Bayer "Macrolon" PC sheet was used in the as-received state. It was found to have residual orientation as a result of the manufacture by extrusion, so all fatigue specimens were cut out in the direction transverse to the extrusion direction. The birefringence of the sheet was measured as $n_E - n_T = 6.2 \times 10^{-4}$, which is quite small. Observation of the birefringence in the TN plane (E extrusion, T transverse, N sheet normal plane) shows that there is a variation from positive on one side to negative on the other, as if caused by a slight residual bending moment, although this birefringence is caused by flow orientation rather than residual stresses.

Bars of length 80 mm and width $w = 10$ mm were milled out from sheet of thickness $d = 6$ mm. The milled surfaces were polished with silicon carbide paper, then 1 and 0.1 μm alumina to remove all visible scratches. They were tested in an Instron TTCM machine using a four-point bending rig, with load cycling between fixed limits. The distances between the inner supports was $2b = 18.8$ mm, and between the inner and other supports was $a = 22.4$ mm. Because PC is a non-linear visco-elastic material, the computation of the surface stresses and strains in the fatigue tests was approximate. A fixed cross-head speed of 10 cm min^{-1} was used, so the cycle time varied between 3 and 7 sec. Determination of the total force $2P_{max}$ (P on each loading point) to form a plastic hinge in the bar, followed by use of the formula

$$\sigma_Y = 4P_{max}\,a/(wd^2)$$

for a non-work-hardening material gave a yield stress of $\sigma_Y = 73$ MN m^{-2} at 20° C. The deflection of the outer loading points relative to the inner ones, Δ, was measured. In general the minimum

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load was 10% of the maximum load, and the load–deflection relation was found to be approximately linear for the fatigue tests. The bending moment decreases linearly between the inner and outer supports, and, for a linearly elastic material, so does the curvature of the beam. On this assumption of linear elasticity, the surface longitudinal strain in the central region of the beam, e, is related to the measured deflection Δ by

$$e = \pm d\Delta/[2a(b + a/3)]. \qquad (1)$$

The fatigue tests were carried out with the maximum surface strain in the range 1.5 to 5%. The nominal surface stress, σ_E, calculated assuming that the PC is a linear elastic material using

$$\sigma_E = 6Pa/(wd^2) \qquad (2)$$

was of the same order or larger than the yield stress, σ_Y. Consequently, the true surface stress must lie between $2/3\,\sigma_E$ (assuming a uniform stress through the section) and σ_Y. The surface strain at which a plastic hinge formed, and the bending moment was a maximum, was found to be 8.3%.

The PVC sheet used was Darvic 025, a transparent unplasticized sheet supplied by ICI Ltd. Single-edge notched specimens of length 100 mm and width 30 mm were cut from 3 mm thick sheet, and a notch 8.5 mm long cut in the centre of one side. An Instron machine was used to fatigue these specimens at 15 cycle min^{-1} between nominal stress limits of 2.5 and 14 MN m^{-2}. Under these conditions a plane stress yielded zone formed directly in front of the notch, so the fatigue crack had to propagate into material which has extended by approximately 100% in the direction of applied tension. Plane stress yielded zones of this type have been described elsewhere [6]. Fatigue crack growth was rapid, with failure occurring in about 1000 cycles.

For both materials the specimens were photographed at intervals during the fatigue process.

3. Results

3.1. Fatigue crack initiation in polycarbonate

Crazes appear on the tensile surface of the PC bars normal to the tensile stress direction. This is not an immediate process: it takes ~ 1000 cycles, and requires a tensile surface strain exceeding 2%. Usually there was a fairly uniform density of up to 200 crazes mm^{-2} in the central region of the

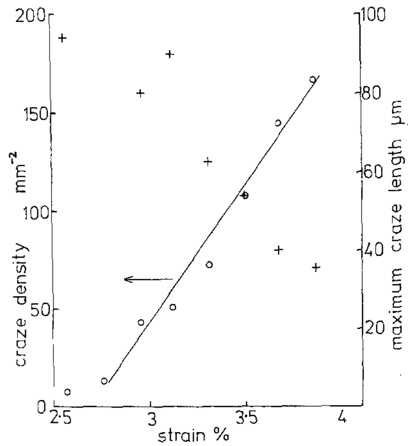

Figure 1 Variation of craze density ○ and maximum craze length + with the maximum surface strain achieved in each fatigue cycle. Measurements taken on a single specimen after 9100 cycles.

bar that has a uniform bending moment applied to it, and the craze density declines to zero at some distance outside the inner supports (Fig. 1). After a further time ~ 1000 cycles shear bands begin to form at the ends of the crazes. Fig. 2 shows a situation where only some of the crazes exhibit shear bands at their ends. The shear bands do not immediately grow to a constant length – they have been observed to grow in length from a particular craze. The histograms of craze sizes (Fig. 3) show that there is wide spread of lengths among the crazes that have no shear bands, but that the crazes that have shear bands have a much more uniform length. This suggests that crazes grow slowly to a certain size then develop shear bands. As the test proceeds a greater and greater fraction of the observed crazes have developed shear bands. The mature craze length depends on the nucleation density; it is about 50 µm in Fig. 3

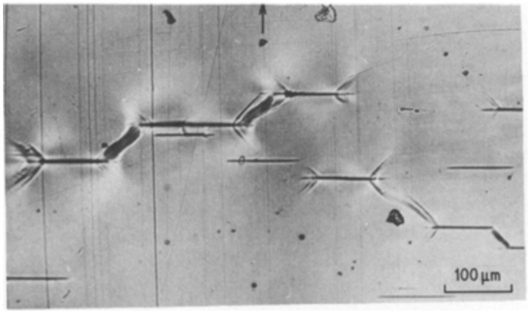

Figure 2 Normal incidence transmitted light photograph of crazes on the tensile surface of a PC specimen. Some crazes have shear bands at their ends. Applied stress was in direction of arrow.

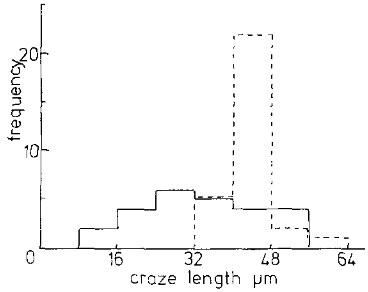

Figure 3 Histogram of craze sises either without – or with – – – shear bands after 1000 cycles. Overall craze density was 81 mm^{-2}.

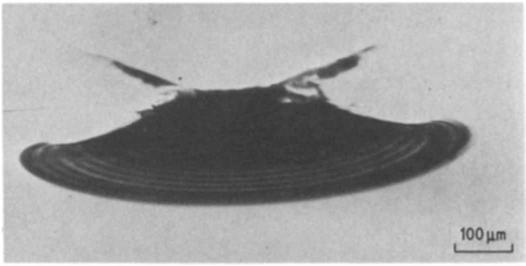

Figure 5 Oblique transmitted light photograph of a surface crack that has developed into an axe-head as it has grown away from the free surface. Fatigue striations are visible near the crack tip.

but it can be 200 μm or more when the surface stress is lower. The ratio of shear band length to craze length can vary from 0.2 to 0.83 in a particular specimen at a particular time.

The appearance of shear bands at the ends of crazes has a number of consequences.

(i) It causes the crazes to open. Oblique observation of the crazes by transmitted light (Fig. 4) shows that there are more thickness fringes on the crazes that have shear bands than those that do not. It does not necessarily convert the crazes into cracks. These appear black since the light is totally internally reflected, and have longer shear bands.

(ii) It prevents further surface growth of the crazes. The stress analysis in a later section shows how the shear bands relieve the stress concentration at the tip of the craze. When fatigue cracks do grow from such crazes they do so by increasing in penetration normal to the free surface. Fig. 5 shows an oblique view of such a crack at a later stage; the crack has not been able to expand in surface length at all at this stage.

(iii) It may link the craze to a neighbouring craze if they have the correct relative orientation. Usually pairs of crazes are linked by a shear band (Fig. 6) but sometimes a series of crazes are linked (as in Fig. 2). This linking of two crazes again aids their opening into cracks – Fig. 4 shows an oblique view where the linked crazes are now cracks.

Crazes only appear to have a limited lifetime (1000s of cycles) under fatigue conditions. Eventually their internal structure breaks down and they become cracks. This may be apparent either from the fact that they cause the total internal reflection of light, or cracks may be directly observable with a scanning electron microscope (Fig. 7). This figure also shows the beginning of fatigue crack propagation along the shear bands.

The main fatigue crack growth usually occurs from an isolated crack, rather than from the mass of small cracks in the central region of the specimen. Thus the fracture plane is usually outside the central supports, and is approximately at the outer limit of crazing. Table I lists the surface stresses and strains calculated from Equations 1 and 2, and the surface strain at the point of fracture e_f if the fracture occurred outside the central region. It can be seen that $e_f = 2.7 \pm 0.1\%$ for the specimens

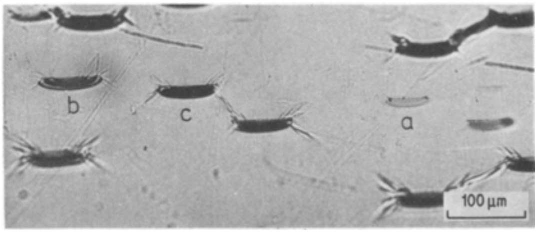

Figure 4 Oblique transmitted light photograph of surface crazes and crack. Crazes A are without shear bands, whereas B has shear bands at both ends. Crazes C have linked up with shear bands and become cracks.

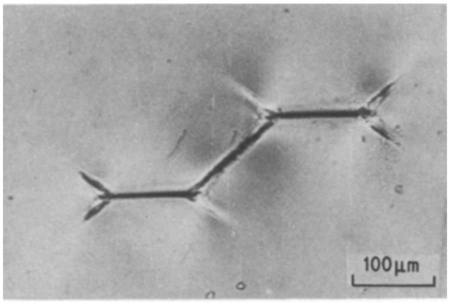

Figure 6 Normal incidence transmitted light photograph of craze pair linked by a shear band at their inner ends.

TABLE I Fatigue failure of PC in four-point bending

Specimen no.	Surface stress σ^E ($MN\,m^{-2}$)	Surface strain e (%)	Strain at failure point (%)	Cycles to failure
1	97	4.3	2.8	2600
2	91	4.1	2.6	9100
3	84	3.6	2.5	6300
4	83	3.9	2.8	5500
5	74	3.0	2.7	3920
6	64	2.6	2.6	15900
7	58	2.2		31000
8	53	2.1		34400
9	45	1.6		> 157000

that failed after forming numerous surface crazes, within 20000 cycles. Failure occurred at a lower maximum strain of ~ 2.2% only when ~ 5 crazes formed on the whole specimen.

The fatigue crack initiation point was either at the side of the tensile surface (presumably because the surface was slightly rougher at this point) or roughly central on the tensile surface of the bar. The fracture surface had three kinds of initial appearance.

(i) A large flat region, with surface length 0.57 mm (specimen 4), or 1.42 mm (specimen 5). It appears that a craze has grown and become a crack without shear bands forming at its ends to restrict the surface growth. This crack grows leaving a featureless fracture surface initially, but when the crack is > 2 mm deep, fatigue striations appear of the type described in [2], presumably one forming for each fatigue cycle.

(ii) A small flat region with shell-like markings (Fig. 8). The surface extent of the craze is in the range 60 to 320 μm, and the striations extend for 660 to 1070 μm into the PC before merging into a flat fracture surface. The initial craze has been limited in surface extent by shear bands, and the crack that forms is too small to grow every cycle. Consequently, a large craze builds up at the crack tip, and only fractures after a number of cycles ~ 1000. This process of crack growth every so often has been observed before [2, 7]. It leads to striations on the fracture surface that are much more widely spaced than the one-per-cycle striations that occur at a later stage. In a specimen when the crack formed in the corner of the cross-section, a number of distinct shear bands are visible when the specimen is viewed along the axis of bending (Fig. 9), and there is a one-to-one correspondence between the shear bands and the fracture surface striations. When the crack grows in the centre of the tensile surface it develops an axe-head shape, expanding in length parallel to the surface.

(iii) For one specimen alone fatigue cracks were observed to grow for some distance down the shear bands, before a crack normal to the tensile stress formed (Fig. 10). It will be seen that initially two crazes have linked, and that the cone of shear bands emanates from them. In this case we have

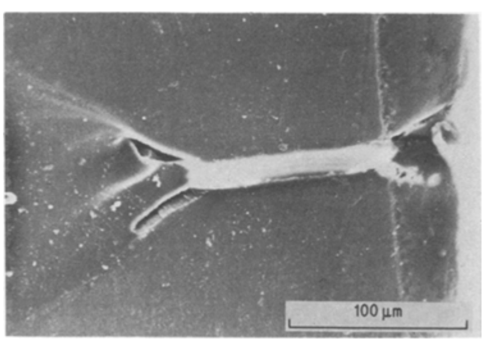

Figure 7 Scanning electron micrograph of a surface crack that has developed from a craze. Note how the crack has propagated some distance down the shear bands which formed at the ends of the original craze.

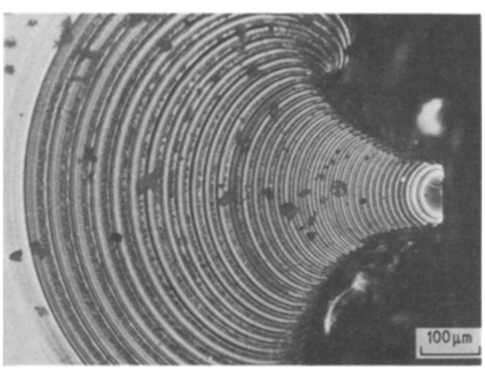

Figure 8 Fracture surface, by normal indicence reflected light, showing shell-like fatigue striations caused by a crack of the type shown in Fig. 5.

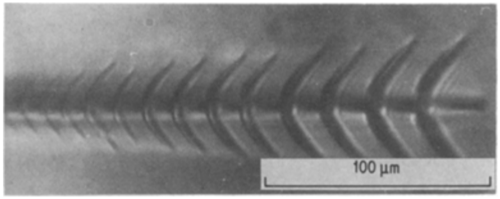

Figure 9 View from a direction parallel to the free surface, and perpendicular to the tensile stress, of the shear bands associated with each fatigue striation of the type shown in Fig. 8. In this case the crack has formed on the corner of the specimen, so the shear bands are close to the side surface.

something similar to the type 1 and type 2 fatigue crack growth in fatigue in polycrystalline metals.

Types (i) and (ii) were observed in proportions 1 :3. The final stages of fatigue crack growth often involved gross yielding of parts of the specimen near the crack i.e. the formation of a plastic hinge on the remaining cross-section. This plastic deformation tended to open up neighbouring cracks.

3.2. Fatigue crack initiation in PVC

In the fatigue tests on PVC it was found that crazes formed on the surface before the material was drawn into the plane stress yielded zone in front of the crack. The effect of the fatigue on crazes in the yielded zone was to cause diffuse shear bands at their ends, and to open them into cracks (Fig. 11). If the main fatigue crack passes close to these surface cracks, then they grow inwards and link up with the main crack (Fig. 12). The conical shape behind the flat original craze area appears on both fracture surfaces, showing that in the yield zone the cracks have opened up by $\sim 50\,\mu m$ as they grow. Detailed examination of the craze fracture surface revealed a characteristic tufted appearance, suggesting that the craze has failed suddenly by opening near its mid plane.

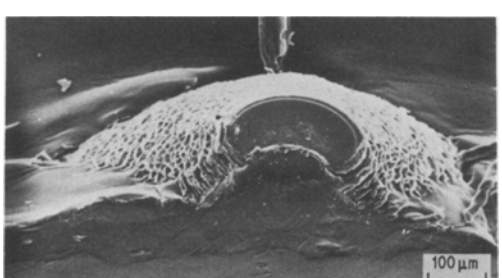

Figure 10 SEM of a fatigued specimen where the craze pair for a distance of 300 µm before propagating on a plane normal the the tensile stress.

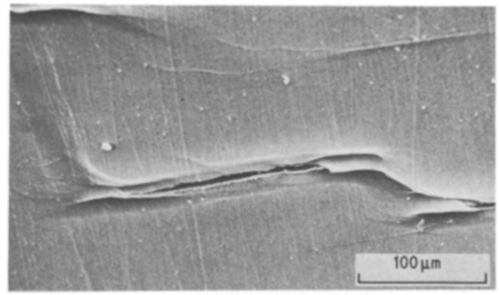

Figure 11 Subsidiary fatigue crack initiation in the yielded zone of the main fatigue crack in PVC. Note the shear bands at the end of the cracks.

4. Stress analysis of the phenomena in fatigue crack initiation

4.1. Energetics of shear band growth

In approaching the stress analysis of craze interaction via shear bands, and of the subsequent crack formation two things must be bourn in mind. Firstly, the stress analysis of isolated crazes is only in its infancy [8, 9, 15], so any values for the stresses across crazes must be treated with caution. Secondly, the surface layer of the bent bar is on the point of general yield, if not already at it, so any stress analysis which assumes that yielding is restricted to certain local regions must only be approximate. Thirdly, both PC and PVC are non-linear viscoelastic materials. However, it is possible to extract useful estimates of changes in the stored elastic energy and of craze opening displacements which can be compared with literature data for PVC and PC.

The problem of a central crack in an elastic sheet with planar yielded zones inclined at angles

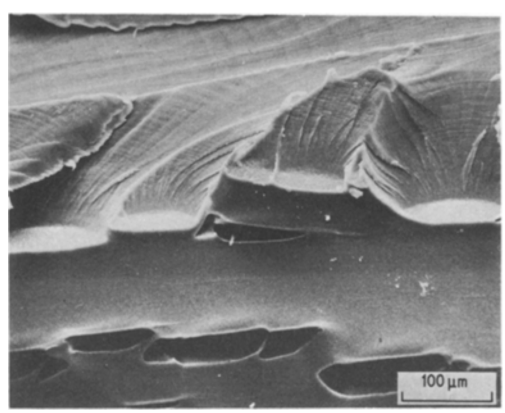

Figure 12 Fatigue fracture surface in PVC showing how the subsidiary fatigue cracks have grown towards the centre of the specimen and linked up with the main crack front.

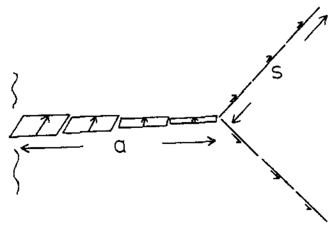

Figure 13 The displacement discontinuity model of a crack plus shear bands. The boundary is divided into ~ 50 straight segments and the stress boundary conditions satisfied at the centre of each.

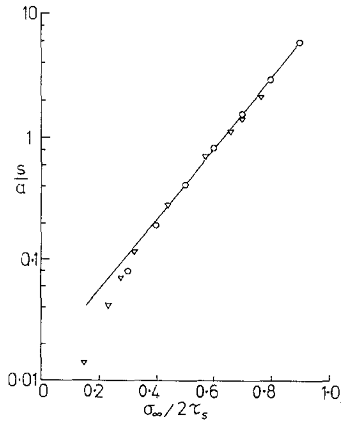

Figure 14 Predicted variation of shear band length s for a central crack of total length $2a$ with the applied tensile stress at infinity σ_∞. τ_S is the shear stress on the shear band. Solid line is Equation 3. ⊙ are predictions of displacement discontinuities calculations, ∇ of [11].

$\pm\theta$ to the ends of the crack was first tackled by Bibly and Swinden [10], but they give few results from their numerical computations. Subsequent more detailed computations [11] over a range of angles θ, and of ratios of the applied tensile stress at infinity σ_∞ to the shear stress on the shear band τ_S, although interpolation is required for particular values of θ and σ_∞/τ_S. We need to make such calculations for more than one craze, but as a preliminary check the method developed in [5] from the work of Crouch [12] was applied to the single crack problem. A number ~ 50 of constant displacement discontinuities over straight segments are distributed along the crack and the shear bands. In the case of the shear bands the displacement discontinuity can only have a component tangential to the segment, but for the crack it can have tangential and normal components (Fig. 13). The stress boundary conditions are only satisfied at the central points of these segments, by solving a set of linear equations relating the stress component at each segment midpoint to the magnitudes of all the displacement discontinuities. In order to find the equilibrium length of the shear bands, these are incremented in length, and the total stored elastic energy U in the surrounding elastic material is calculated for each shear band length, using the formula

$$U = \sum_{i=1}^{n} \tfrac{1}{2} a_i(\sigma_{Ni}\delta_{Ni} + \sigma_{Ti}\delta_{Ti})$$

where a_i is the length of, σ_{Ni} and σ_{Ti} are the normal and tangential stress components at the centre of, δ_{Ni} and δ_{Ti} are the normal and shear displacement discontinuities of, the ith segment. To avoid any contribution from any distant loading system the problem solved is that of an internally pressurized crack in a sheet that is stress free at infinity, the internal pressures being chosen so that when a uniform stress field of $\sigma_{yy} = \sigma_\infty$ everywhere is superimposed the original problem is restored. It was found in [5] that when U was maximized with respect to the yielded zone length for the problem of a strip yielded zone ahead of a central crack, this gave the same yielded zone length as Dugdale calculated [13] using a conformal transformation method. Hence it was assumed that the same maximization of U would give the equilibrium shear band length for this problem, for a constant applied stress at infinity. Note that this method is not the same as the calculation of Palmer and Rice's J integral, which he has used to analyse shear band propagation in clay [14]. It can be shown that for a central crack with four symmetrical shear bands the J integral evaluated for a path from the upper to the lower crack surface would reduce to

$$J = 4\tau_S\delta_{Tc}$$

where δ_{Tc} is the shear displacement discontinuity of the shear band at the end of the shear band which joins the crack.

Fig. 14 shows how the equilibrium shear band length, S, calculated from the displacement discontinuities method varies with applied tensile stress. The results agree well with the empirical relationship used by Vitek to fit his computations [11]. The points in Fig. 14 fall on a straight line of the equation

$$s/a = 0.015 \exp\left[6.642(\sigma_\infty/2\tau_S)\right] \qquad (3)$$

for $0.4 < \sigma_\infty/2\tau_S < 0.9$.

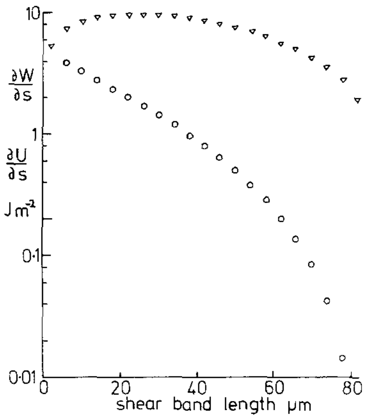

Figure 15 ○ variation in the rate of increase of stored elastic energy as the shear band length s approaches its equilibrium value for a craze of total length $200\,\mu m$, $\sigma_\infty - \sigma_c = 30\,MN\,m^{-2}$, $2\tau_S - \sigma_c = 50\,Mn\,m^{-2}$. The variation in plastic energy dissipation rate $\partial w/\partial s$ is also shown, ∇.

The crack plus shear band model can readily be adapted to a craze plus shear bands, if it is assumed that the craze transmits a uniform normal stress σ_c. An addition of a uniform tensile stress σ_c normal to the crack in the crack plus shear band model does not modify the analysis but merely requires that $\sigma_\infty/2\tau_S$ in Equation 3 is replaced by $(\sigma_\infty - \sigma_c)/(2\tau_S - \sigma_c)$.

The incremental method of computing the equilibrium shear band length provides data on how the stored elastic energy U in the elastic material surrounding the craze and shear bands changes with the shear band lengths (because of the symmetrical growth of four shear bands from the craze, U is calculated for a quarter plane). Fig. 15 shows how $\partial U/\partial s$ varies with s for a particular set of stress parameters. The initial value of $5\,J\,m^{-2}$ decreases to below $1\,J\,m^{-2}$ when the shear band has grown to 50% of its equilibrium length. If the craze was increased in size by a scaling factor X, the value of $\partial U/\partial s$ would be X times larger if the other parameters were unchanged. If the craze size is kept constant and the stresses $\sigma_\infty - \sigma_c$ and $2\tau_S - \sigma_c$ increased by a factor Y the value of $\partial U/\partial s$ increases by a factor Y^2. However, the parameters chosen are appropriate to the experimental observations. Fig. 15 also shows how the rate of plastic energy dissipation in the shear band $\partial w/\partial s$ varies with shear band length. w is calculated by summation, for the displacement discontinuities along the shear band only, using

$$w = \sum_i a_i \tau_S \delta_{Ti}.$$

It can be seen that $\partial w/\partial s$ is always larger than $\partial U/\partial s$ and the ratio of the sizes of the two quantities increases as the shear band grows. Thus the reduction in the potential energy of the system of the specimen and the loading mechanism, which is equal to $\partial U/\partial s$ for the specimen when the loads at infinity are kept constant, is always smaller than the plastic energy dissipation rate. Hence the shear bands cannot grow spontaneously.

The structure of a shear band has some similarities with that of a deformation twin in a metal crystal, in that a region with a shear strain ~ 1 is contained between closely spaced parallel planes. Hence there will be a surface energy $\gamma_s\,J\,m^{-2}$ associated with each surface of the shear band; and the magnitude of this probably lies in the range 0.1 to $10\,J\,m^{-2}$. If we postulate that a critical $\partial U/\partial s$ is required to nucleate a shear band, then we can deduce that crazes must reach a critical length (and possibly a critical stage in their break down by fatigue which reduces σ_c) before it is possible for shear bands to be nucleated. To explain the observation that cracks smaller than $32\,\mu m$ in length do not exhibit shear bands (Fig. 3) then value of the critical $\partial U/\partial s$ is $4\,J\,m^{-2}$ for the parameters $\sigma_\infty = 70\,MN\,m^{-2}$, $\tau_S = 58\,MN\,m^{-2}$.

4.2. Craze opening as a result of shear band growth

In order to investigate whether shear band growth would encourage the breakdown of crazes, by extending the craze material beyond some limit, calculations were made of the changes in the maximum craze opening displacement using the displacement discontinuity method.

Applied stresses were chosen that were similar to those in the fatique experiments, and the tensile stress across the craze was chosen to give a shear band length typical of those found experimentally, given that the shear stress on the shear band was similar to the shear yield stress of PC or PVC. Fig. 16a shows the initial situation of a $200\,\mu m$ long two-dimensional craze, transmitting a stress $\sigma_c = 30\,MN\,m^{-2}$. The calculated central opening is $3.30\,\mu m$ when the applied stress is $60\,MN\,m^{-2}$ (the exact value from elasticity theory is $3.19\,\mu m$). When shear bands are allowed to grow to their equilibrium length of $84\,\mu m$ (Fig. 16b) against a shear stress $\tau_S = 25\,MN\,m^{-2}$, the central craze opening increases to $3.51\,\mu m$. This removes the stress concentration at the craze tip (the stress intensity factor K_I was $0.53\,MN\,m^{-3/2}$), but

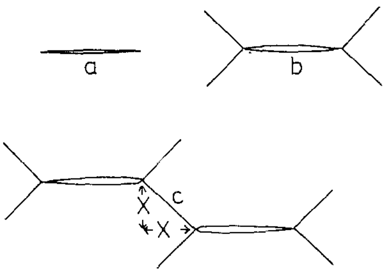

Figure 16 Modelling of the interactions of two crazes having shear bands at their ends. (a) isolated craze, (b) isolated craze with equilibrium length shear bands, (c) two crazes plus shear bands. X is the separation of the inner craze tips.

hardly increases the craze opening displacement. Next the effect of a neighbouring craze in a position suitable for shear band link up (Fig. 16c) was investigated. The separation X of the two inner craze tips was varied and the shear bands pointing towards the origin allowed to grow to their equilibrium length (the other shear bands have a constant 84 μm length). Table II shows the results.

It is apparent that closely spaced pairs of crazes, with a $\pm 45°$ relative orientation of their inner craze tips, can lead to significant increases in the maximum craze opening displacement.

5. Discussion

Firstly, the phenomena observed in the fatigue crack initiation experiments will be compared with the known behaviour of glassy polymers in other circumstances, then the theoretical analyses and predictions of behaviour will be compared critically with the observations. As far as we know no other observations of shear bands emanating from crazes or cracks have been reported during fatigue tests on polymers.

The observation that shear bands only form when the crazes (cracks) exceed 32 μm in surface length lead to the postulate that the rate of change of stored elastic energy must exceed 4 J m^{-2} to initiate a shear band. The shear bands are to some extent a surface phenomena – that is they form in the layer that extends approximately 50 μm from the free surface. It would be expected, on the basis of the stress intensity factors alone, that the shear bands should form at every point of the circumference of the semi-circular surface crack (Fig. 8). They actually form where the crack meets the free surface, and at no other points. The reason for this may be connected with the observation that in polystyrene, coarse shear bands appear at ridges on the free surface [15].

TABLE II Calculations of craze opening for craze pairs

x (and y) separation of inner craze tips X (μm)	Do shear bands link up at origin?	Maximum craze opening (μm)
300	no	3.63
200	yes	3.81
100	yes	4.49
40	yes	5.09

It has not been possible to use the observed ratios of shear band length S to crack length a, to produce precise values, from the analysis of Fig. 14, of the shear stress active on the shear bands. This is for several reasons. If the craze has not become a crack then there are two unknowns σ_{cr} and τ_S and only one known quantity S/a. Even when it is known via oblique observation that a crack has formed there are a range of values of S/a observed in any specimen at a particular time; presumably because the growth of shear band length is a slow process. Using the range of 0.12 to 0.83 for S/a (Fig. 14), and the value of σ_∞ equal to the observed yield stress of 73 MN m^{-2}, gives a range of values for τ_S from 94 to 58 MN m^{-2}. These are rather high values, and lend support to the idea that the shear bands grow slowly towards their equilibrium lengths under fatigue loading. The analysis cannot proceed until the kinetics of shear band growth are studied further. Since the analysis presented in this paper is for monotonic loading, whereas the experiments used fatigue loading, observations should also be made under static loading.

It has been possible to show by analysis that the process of a shear band forming between two close craze tips of the correct relative orientation can lead to a significant increase in the craze opening displacement (Table II). These values can be compared with values observed for the craze opening at a crack tip in PC. Frazer and Ward [16] found that the critical craze opening displacement was 5.8 μm for Lexan PC at 22.5° C under sustained loading. This is close enough to the values in Table II for the craze tips separated by 40 μm to suggest that the shear band formation could cause the critical craze opening displacement to be exceeded, and hence a crack to form. Certainly experimental observations show that this can happen.

In recent review, Schultz [17] mentions that most workers have found a one-to-one correlation between fatigue fracture surface striations and fatigue cycles for PC. In another paper [18] he shows that this is not the case in the early stages of fatigue crack growth. We have confirmed that many fatigue cycles are necessary to advance the crack by one increment in the early stage of fatigue crack growth. It is of interest to calculate the stress intensity factor range ΔK involved. Using the formulae for a semicircular surface crack [19] of radius $30\,\mu m$, when the applied stress range is equal to the yield stress ($73\,MN\,m^{-2}$), gives a ΔK of $0.44\,MN\,m^{-3/2}$. For a crack of radius $160\,\mu m$ the $\Delta K = 1.0\,MN\,m^{-3/2}$. By reference to published data for the fatigue crack growth rate of PC [2] the corresponding average growth rates per cycle should be 0.02 and $0.3\,\mu m\,cycle^{-1}$, respectively. These values appear to be similar to those we observed, so we can conclude that normal fatigue crack growth is occurring at this stage.

At the beginning of this paper we posed the question whether it was the largest craze that grew into the crack that caused fracture, irrespective of its position relative to other crazes, or whether craze interactions were most important in initiating fracture. From the experimental observations we conclude that both factors are involved. Crazes can interact readily even when shear bands do not form, and a recent analysis [5] shows that this restricts the surface growth, and reduces the rate of penetration away from the free surface. Hence the largest crazes will grow in regions where the surface density is very low – here at the point beyond the inner four-point bending load points where the surface strain drops to 2.7%. Consequently, craze interactions determine the position of the fracture site, and it is the largest craze that appears in an area of very low craze density that grows into the crack that causes fracture.

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